

ESPRESSIONI CON PRODOTTI NOTEVOLI

RACCOLTA DI ESERCIZI SVOLTI

Lorenzo Andreassi

www.lorenzoandreassi.it

Esercizio 1

$$(a - 2b)^3 + (2a - b)^3 - 9(a - b)^3 =$$

$$a^3 + 3(a)^2(-2b) + 3(a)(-2b)^2 - 8b^3 + 8a^3 + 3(2a)^2(-b) + 3(2a)(-b)^2 +$$
$$- b^3 - 9 \cdot [a^3 + 3(a)^2(-b) + 3(a)(-b)^2 - b^3] =$$

$$a^3 - 6a^2b + 12ab^2 - 8b^3 + 8a^3 - 12a^2b + 6ab^2 - b^3 +$$
$$- [9a^3 + 27(a)^2(-b) + 27(a)(-b)^2 - 9b^3] =$$

$$9a^3 - 18a^2b + 18ab^2 - 9b^3 - 9a^3 + 27a^2b - 27ab^2 + 9b^3 =$$

$$-18a^2b + 18ab^2 + 27a^2b - 27ab^2 =$$

$$-9a^2b + 9ab^2.$$

Esercizio 2

$$(5a^2 + 2ab - c^2)(5a^2 - 2ab + c^2) + \left(2ab - \frac{1}{2}c^2\right)^2 =$$
$$= [5a^2 + (2ab - c^2)] \cdot [5a^2 - (2ab - c^2)] + 4a^2b^2 - 2abc^2 + \frac{1}{4}c^4 =$$
$$= 25a^4 - (2ab - c^2)^2 + 4a^2b^2 - 2abc^2 + \frac{1}{4}c^4 =$$
$$= 25a^4 - (4a^2b^2 - 4abc^2 + c^4) + 4a^2b^2 - 2abc^2 + \frac{1}{4}c^4 =$$
$$= 25a^4 - 4a^2b^2 + 4abc^2 - c^4 + 4a^2b^2 - 2abc^2 + \frac{1}{4}c^4 =$$
$$= 25a^4 + 2abc^2 + \frac{-4+1}{4}c^4 =$$
$$= 25a^4 + 2abc^2 - \frac{3}{4}c^4$$

Esercizio 3

$$\begin{aligned} & \left(a^2 + \frac{1}{2}b\right)^2 - \left(a^2 - \frac{1}{2}b\right)^2 - 2a^2b - 8a^2b^4 : (-4ab^2) = \\ & = a^4 + a^2b + \frac{1}{4}b^2 - a^4 + a^2b - \frac{1}{4}b^2 - 2a^2b + 2ab^2 = \\ & = +a^2b + a^2b - 2a^2b + 2ab^2 = \\ & + 2ab^2 \end{aligned}$$

Esercizio 4

$$\begin{aligned} & \left[\left(2ab - \frac{1}{2}b^2\right)(3a + 2b) \left(-\frac{1}{3}b\right) - \frac{1}{3}b^4 + \frac{5}{6}ab^3 \right] : \left(\frac{1}{3}a^2b^2\right) \\ & = \left[\left(6a^2b + 4ab^2 - \frac{3}{2}ab^2 - b^3\right) \left(-\frac{1}{3}b\right) - \frac{1}{3}b^4 + \frac{5}{6}ab^3 \right] : \left(\frac{1}{3}a^2b^2\right) = \\ & = \left[-2a^2b^2 - \frac{4}{3}ab^3 + \frac{1}{2}ab^3 + \frac{1}{3}b^4 - \frac{1}{3}b^4 + \frac{5}{6}ab^3 \right] : \left(\frac{1}{3}a^2b^2\right) = \\ & = (-2a^2b^2) : \left(\frac{1}{3}a^2b^2\right) = -6 \end{aligned}$$

Esercizio 5

$$\begin{aligned} & \left(\frac{1}{2}a^2b - a\right) \left(\frac{1}{2}a^2b + a\right) : \left(\frac{1}{2}a\right) \left(\frac{1}{2}a^2b^2 + 2\right) \left(3a - \frac{5}{2}a + \frac{1}{2}a\right) + 4a^2 \\ & = \left(\frac{1}{4}a^4b^2 - a^2\right) : \left(\frac{1}{2}a\right) \left(\frac{1}{2}a^2b^2 + 2\right) (a) + 4a^2 = \\ & = \left(\frac{1}{2}a^3b^2 - 2a\right) \left(\frac{1}{2}a^2b^2 + 2\right) a + 4a^2 = \left(\frac{1}{4}a^5b^4 - 4a\right) a + 4a^2 = \\ & = \frac{1}{4}a^6b^4 - 4a^2 + 4a^2 = \frac{1}{4}a^6b^4 \end{aligned}$$

Esercizio 6

$$\begin{aligned} & \left(4a^3 - \frac{1}{2}a^2\right) \left(\frac{1}{2}a + 4\right) + 2a(-a^3 + a) \\ & = 2a^4 + 16a^3 - \frac{1}{4}a^3 - 2a^2 - 2a^4 + 2a^2 = \frac{63}{4}a^3 \end{aligned}$$

Esercizio 7

$$\begin{aligned} & (x - y)(2x - 1) - x(2x - 1) - y(1 + 2x) \\ & = 2x^2 - x - 2xy + y - 2x^2 + x - y - 2xy = -4xy \end{aligned}$$

Esercizio 8

$$[3a(a^2 - 1) - 2a(2a - 1)] : 2a$$

$$= [3a^3 - 3a - 4a^2 + 2a] : 2a = (3a^3 - a - 4a^2) : 2a = \frac{3}{2}a^2 - \frac{1}{2} - 2a$$

Esercizio 9

$$-\frac{1}{2}xy \left[\left(\frac{3}{4}x^2y^2 + \frac{1}{2}x^2y^2 \right) : (-2x^2) \right] + \frac{7}{8}xy^3 = -\frac{1}{2}xy \left[\left(\frac{5}{4}x^2y^2 \right) : (-2x^2) \right] + \frac{7}{8}xy^3 =$$
$$= -\frac{1}{2}xy \left(-\frac{5}{8}y^2 \right) + \frac{7}{8}xy^3 = \frac{5}{16}xy^3 + \frac{7}{8}xy^3 = \frac{19}{16}xy^3$$

Esercizio 10

$$(2x - 1)^2 - (1 + 2x)(1 - 2x) - 4x(2x - 1)$$

$$= 4x^2 - 4x + 1 - (1 - 4x^2) - 8x^2 + 4x = 4x^2 - 4x + 1 - 1 + 4x^2 - 8x^2 + 4x = 0$$

Esercizio 11

$$(x - 1)^3 - (x + 1)^2 - (x + 1)(-x + 1) - (x^2 + 1)(x - 3)$$

$$= x^3 - 3x^2 + 3x - 1 - (x^2 + 2x + 1) - (1 - x^2) - (x^3 - 3x^2 + x - 3) =$$

$$= x^3 - 3x^2 + 3x - 1 - x^2 - 2x - 1 - 1 + x^2 - x^3 + 3x^2 - x + 3 = 0$$

Esercizio 12

$$(x - 3)(x - 2) + (x + 3)(x - 2) - 2(x - 3)(x + 2) - 12$$

$$= x^2 - 5x + 6 + x^2 + x - 6 - 2(x^2 - x - 6) - 12 =$$

$$= x^2 - 5x + 6 + x^2 + x - 6 - 2x^2 + 2x + 12 - 12 = -2x$$

Esercizio 13

$$[(x - 2)(x + 2) - (x - 1)^3 - 4x^2]^2 - (x^3 - 3)^2 - 6x(x^3 + 2x + 3) + 3x^2$$

$$= [x^2 - 4 - (x^3 - 3x^2 + 3x - 1) - 4x^2]^2 - (x^6 - 6x^3 + 9) - 6x^4 - 12x^2 - 18x + 3x^2 =$$

$$= [x^2 - 4 - x^3 + 3x^2 - 3x + 1 - 4x^2]^2 - x^6 + 6x^3 - 9 - 6x^4 - 12x^2 - 18x + 3x^2 =$$

$$= (-3 - x^3 - 3x)^2 - x^6 + 6x^3 - 9 - 6x^4 - 9x^2 - 18x =$$

$$= 9 + x^6 + 9x^2 + 6x^3 + 18x + 6x^4 - x^6 + 6x^3 - 9 - 6x^4 - 9x^2 - 18x = 12x^3$$